Fermions on Thick Branes in the Background of Sine-Gordon Kinks

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Abstract

A class of thick branes in the background of sine-Gordon kinks with a scalar potential $V(\phi) = p(1 + \cos\frac{2\phi}{q})$ was constructed by R. Koley and S. Kar [Classical Quantum Gravity **22**, 753 (2005)]. In this paper, in the background of the warped geometry, we investigate the issue of localization of spin half fermions on these branes in the presence of two types of scalar-fermion couplings: $\eta\bar{\Psi}\phi\Psi$ and $\eta\bar{\Psi}\sin\phi\Psi$. By presenting the mass-independent potentials in the corresponding Schrödinger equations, we obtain the lowest Kaluza–Klein (KK) modes and a continuous gapless spectrum of KK states with $m^2 > 0$ for both types of couplings. For the Yukawa coupling $\eta\bar{\Psi}\phi\Psi$, the effective potential of the right chiral fermions for positive q and η is always positive, hence only the effective potential of the left chiral fermions could trap the corresponding zero mode. This is a well-known conclusion which had been discussed extensively in the literature. However, for the coupling $\eta\bar{\Psi}\sin\phi\Psi$, the effective potential of the right chiral fermions for positive q and η is no longer always positive. Although the value of the potential at the location of the brane is still positive, it has a series of wells and barriers on each side, which ensures that the right chiral fermion zero mode could be trapped. Thus we may draw the remarkable conclusion: for positive η and q, the potentials of both the left and right chiral fermions could trap the corresponding zero modes under certain restrictions.

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I. INTRODUCTION

The suggestion that extra dimensions may not be compact [1, 2, 3, 4, 5, 6, 7, 8] or large [9, 10] can provide new insights for solving gauge hierarchy problem [10] and cosmological constant problem [4, 7, 11], etc. In the framework of brane scenarios, gravity is free to propagate in all dimensions, while all the matter fields are confined to a 3–brane [4, 6, 10, 12, 13]. In Ref. [1], an alternative scenario of the compactification had been proposed. In this scenario, the internal manifold does not need to be compactified to the Planck scale any more, which is one of reasons why this new compactification scenario has attracted so much attention. Among all of the brane world models, there is an interesting and important model in which extra dimensions comprise a compact hyperbolic manifold [8]. The model is known to be free of usual problems that plague the original Arkani-Hamed-Dimopoulos-Dvali (ADD) models and share many common features with Randall-Sundrum (RS) models.

In the brane world scenario, an important question is localization of various bulk fields on a brane by a natural mechanism. It is well known that massless scalar fields [14] and graviton [1] can be localized on branes of different types, and that spin 1 Abelian vector fields can not be localized on the RS brane in five dimensions, but can be localized in some higher-dimensional cases [15]. Spin 1/2 fermions do not have normalizable zero modes and hence can not be localized in five and six dimensions [14, 15, 16, 17, 18, 19, 20].

Recently, an increasing interest has been focused on the study of thick brane scenarios based on gravity coupled to scalars in higher dimensional space-time [21, 22, 23, 24, 25, 26, 27]. A virtue of these models is that the branes can be obtained naturally rather than introduced by hand [21]. Besides, these scalar fields provide the "material" of which the thick branes are made. In Ref. [28], exact solutions of the Einstein–scalar equations with a sine-Gordon potential and a negative cosmological constant were constructed. In this system the scalar field configuration in fact is a kink, which provides a thick brane realization of the brane world as a domain wall in the bulk. The warped background space-time has a non–constant but asymptotically negative Ricci curvature. Such a configuration was also illustrated in several examples in the literature [17, 18].

The localization problem of spin half fermions on thick branes is interesting and important. Localization of fermions in general space-times had been studied for example in [29]. In five dimensions, with the scalar–fermion coupling, there may exist a single bound state

and a continuous gapless spectrum of massive fermion KK states [30, 31, 32, 33], while for some other brane models, there exist finite discrete KK states (mass gap) and a continuous gapless spectrum starting at a positive m^2 [34, 35]. In Ref. [36], it was found that fermions can escape into the bulk by tunnelling, and the rate depends on the parameters of the scalar potential. In Ref. [28], the authors obtained trapped discrete massive fermion states on the brane, which in fact are quasi-bound and have a finite probability of escaping into the bulk. It is also interesting to note that in Ref. [37] fermion modes in a sine-Gordon kink and kink-anti-kink system were also studied in some 1+1 dimensional scalar field theories. It was shown that there exist discrete bound states. However, when the wall and anti-wall approach each other, the system cannot support fermion bound states and the discrete states merge into the continuous spectrum of the Dirac equation.

It is known that, under the Yukawa coupling $\eta\bar{\Psi}\phi\Psi$, only one of the effective potentials of the left and right chiral fermions could trap the corresponding zero mode for positive q and η. In this paper, we will reinvestigate the localization issues of fermions on the branes obtained in Ref. [28] in the presence of different types of scalar-fermion couplings, by presenting the mass-independent potentials in the corresponding Schrödinger equations. We will show that, for the coupling $\eta\bar{\Psi}\sin\phi\Psi$, not only the potential of the left chiral fermions but also the potential of the right ones could trap the corresponding zero modes for positive η and q under certain different restrictions for each case. Besides, instead of discrete massive KK mode, there exists a continuous gapless spectrum of KK states with $m^2 > 0$. The shapes of the potentials also suggest that the massive KK modes asymptotically turn into plane waves, which represent delocalized massive KK fermions.

The paper is organized as follows: In section II, we first give a brief review of the thick brane arising from a sine-Gordon potential in a 5-dimensional space-time. Then, in section III, we study localization of spin half fermions on the thick brane with two different types of scalar-fermion interactions by presenting the shapes of the potentials of the corresponding Schrödinger problem. Finally, a brief conclusion and discussion are presented.

II. REVIEW OF THE SINE-GORDON KINK AND THE THICK BRANES

Let us consider thick branes arising from a real scalar field with a sine-Gordon potential

$$V(\phi) = p\left(1 + \cos\frac{2\phi}{q}\right). \tag{1}$$

This special potential was considered in Ref. [28] and other different choices of $V(\phi)$ can be found in the work of others [22]. In the model, the bulk sine-Gordon potential provides a thick brane realization of the Randall–Sundrum scenario, and the soliton configuration of the scalar field dynamically generate the domain wall configuration with warped geometry. The action for such a system is given by

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2\kappa_5^2}(R - 2\Lambda) - \frac{1}{2}g^{MN}\partial_M\phi\partial_N\phi - V(\phi)\right], \tag{2}$$

where $\kappa_5^2 = 8\pi G_5$ with G_5 the 5-dimensional Newton constant, and Λ is the 5-dimensional cosmological constant. The line-element which results in a 4-dimensional Poincaré invariance of the action (2) is assumed as

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{3}$$

where $e^{2A(y)}$ is the warp factor and y stands for the extra coordinate. The scalar field is considered to be a function of y only. The field equations, that are derivable from (2) with the ansatz (3), reduce to the following coupled nonlinear differential equations

$$A'' = -\frac{\kappa_5^2}{3}\phi'^2, \tag{4}$$

$$A'^2 = -\frac{\kappa_5^2}{12}\left(\phi'^2 - 2V\right) + \frac{\Lambda}{6}, \tag{5}$$

$$\phi'' + 4A'\phi' = \frac{dV}{d\phi}. \tag{6}$$

For the sine-Gordon potential (1), the solution can be calculated [28]:

$$A(y) = -\tau\ln\cosh ky, \tag{7}$$

$$\phi(y) = 2q\arctan\left(\exp ky\right) - \frac{\pi q}{2}, \tag{8}$$

where τ and k are given by

$$\tau = \frac{1}{3}\kappa_5^2 q^2, \quad k = \frac{\sqrt{6|\Lambda|}}{6\tau}. \tag{9}$$

The parameters q and Λ are free to choose, and p is given by

$$p = \frac{|\Lambda|}{2\kappa_5^4} \left(\frac{\kappa_5^2}{3} + \frac{1}{4q^2} \right). \tag{10}$$

Observing the forms of (7) and (8), one can find that the configuration of $\phi(y)$ is a kink for positive q, and the warp factor $A(y)$ is a smooth function. Besides these properties, more detailed discussions can be found in Ref. [28].

The extensive work had been done on non-supersymmetric [21, 23, 24, 38] as well as supersymmetric [39, 40, 41] domain walls in different models. In Ref. [40], Maru *et al* constructed an analytic non-Bogomol'nyi-Prasad-Sommerfeld (BPS) solution of the sine-Gordon domain wall in a 4-dimensional global supersymmetric model. In Ref. [41], this sine-Gordon domain wall solution was extended to a solution in 4-dimensional supergravity and its stability had been examined. In these papers, although sine-Gordon domain wall had been considered in supersymmetric theories, the properties of the solution itself are also valid in the purely bosonic sector. In the following, we will reconsider the issue of localization of spin half fermions on the 3–brane in the presence of two types of kink-fermion couplings in the background of the sine-Gordon kink (8) and the corresponding warped geometry.

III.   LOCALIZATION OF FERMIONS ON THE THICK BRANES

Now, let us investigate whether spin half fermions can be localized on the brane. We will analyze the spectrum of fermions for the thick branes by presenting the mass-independent potentials in the corresponding Schrödinger equations. In order to get the mass-independent potentials, we will follow Ref. [1] and change the metric given in (3) to a conformally flat one

$$ds_5^2 = \mathrm{e}^{2A} \left(\eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right) \tag{11}$$

by performing the coordinate transformation

$$dz = \mathrm{e}^{-A(y)} dy. \tag{12}$$

In five dimensions, fermions are four component spinors and their Dirac structure is described by $\Gamma^M = e_{\bar{M}}^M \Gamma^{\bar{M}}$ with $\{\Gamma^M, \Gamma^N\} = 2g^{MN}$. In this paper, $\bar{M}, \bar{N}, \cdots$ denote the local Lorentz indices, and $\Gamma^{\bar{M}}$ are the flat gamma matrices in five dimensions. In our set-up,

$\Gamma^M = (\mathrm{e}^{-A}\gamma^\mu, \mathrm{e}^{-A}\gamma^5)$, where γ^μ and γ^5 are the usual flat gamma matrices in the Dirac representation. The Dirac action of a massless spin $1/2$ fermion coupled to the scalar is

$$S_{1/2} = \int d^5x\sqrt{-g}\left(\bar{\Psi}\Gamma^M D_M\Psi - \eta\bar{\Psi}F(\phi)\Psi\right), \tag{13}$$

where the covariant derivative D_M is defined as $D_M\Psi = (\partial_M + \frac{1}{4}\omega_M^{\bar{M}\bar{N}}\Gamma_{\bar{M}}\Gamma_{\bar{N}})\Psi$ with the spin connection $\omega_M = \frac{1}{4}\omega_M^{\bar{M}\bar{N}}\Gamma_{\bar{M}}\Gamma_{\bar{N}}$ and

$$\begin{aligned}
\omega_M^{\bar{M}\bar{N}} &= \frac{1}{2}e^{N\bar{M}}(\partial_M e_N^{\bar{N}} - \partial_N e_M^{\bar{N}}) \\
&- \frac{1}{2}e^{N\bar{N}}(\partial_M e_N^{\bar{M}} - \partial_N e_M^{\bar{M}}) \\
&- \frac{1}{2}e^{P\bar{M}}e^{Q\bar{N}}(\partial_P e_{Q\bar{R}} - \partial_Q e_{P\bar{R}})e_M^{\bar{R}}.
\end{aligned} \tag{14}$$

With the metric (11), the non-vanishing components of the spin connection ω_M are

$$\omega_\mu = \frac{1}{2}(\partial_z A)\gamma_\mu\gamma_5. \tag{15}$$

Then the equation of motion is given by

$$\left\{\gamma^\mu\partial_\mu + \gamma^5\left(\partial_z + 2\partial_z A\right) - \eta\,\mathrm{e}^A F(\phi)\right\}\Psi = 0, \tag{16}$$

where $\gamma^\mu\partial_\mu$ is the Dirac operator on the brane.

Now we study the above 5-dimensional Dirac equation. From the equation of motion (16), we will search for the solutions of the general chiral decomposition

$$\Psi(x, z) = \sum_n \psi_{Ln}(x)\alpha_{Ln}(z) + \sum_n \psi_{Rn}(x)\alpha_{Rn}(z) \tag{17}$$

with $\psi_{Ln}(x) = -\gamma^5\psi_{Ln}(x)$ and $\psi_{Rn}(x) = \gamma^5\psi_{Rn}(x)$ the left-handed and right-handed components of a 4-dimensional Dirac field, the sum over n can be both discrete and continuous. Here, we assume that $\psi_L(x)$ and $\psi_R(x)$ satisfy the 4-dimensional massive Dirac equations $\gamma^\mu\partial_\mu\psi_{Ln}(x) = m_n\psi_{R_n}(x)$ and $\gamma^\mu\partial_\mu\psi_{Rn}(x) = m_n\psi_{L_n}(x)$. Then $\alpha_L(z)$ and $\alpha_R(z)$ satisfy the following coupled equations

$$\left\{\partial_z + 2\partial_z A + \eta\,\mathrm{e}^A F(\phi)\right\}\alpha_{Ln}(z) = m_n\alpha_{Rn}(z), \tag{18a}$$

$$\left\{\partial_z + 2\partial_z A - \eta\,\mathrm{e}^A F(\phi)\right\}\alpha_{Rn}(z) = -m_n\alpha_{Ln}(z). \tag{18b}$$

In order to obtain the standard 4-dimensional action for the massive chiral fermions, we need the following orthonormality conditions for α_{L_n} and α_{R_n}:

$$\int_{-\infty}^{\infty} \mathrm{e}^{4A}\alpha_{Lm}\alpha_{Rn}dz = \delta_{LR}\delta_{mn}. \tag{19}$$

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Defining $\widetilde{\alpha}_L = e^{2A}\alpha_L$, we get the Schrödinger-like equation for the left chiral fermions

$$[-\partial_z^2 + V_L(z)]\widetilde{\alpha}_{Ln} = m_n^2\widetilde{\alpha}_{Ln}, \tag{20}$$

where the effective potential is given by

$$V_L(z) = e^{2A}\eta^2 F^2(\phi) - e^A\eta\,\partial_z F(\phi) - (\partial_z A)e^A\eta F(\phi). \tag{21}$$

For the right chiral fermions, the corresponding potential can be written out easily by replacing $\eta \to -\eta$ from above potential

$$V_R(z) = e^{2A}\eta^2 F^2(\phi) + e^A\eta\,\partial_z F(\phi) + (\partial_z A)e^A\eta F(\phi). \tag{22}$$

It can be seen clearly that, for the left (right) chiral fermion localization, there must be some kind of Yukawa coupling. This situation can be compared with the one in the RS framework [14], where additional localization method [42] was introduced for spin 1/2 fields. Furthermore, $F(\phi(z))$ must be an odd function of $\phi(z)$ when we demand that $V_L(z)$ or $V_R(z)$ is Z_2-even with respect to the extra dimension z. In this paper, we consider two cases $F(\phi(z)) = \phi(z)$ and $F(\phi(z)) = \sin\phi(z)$ as examples. For each case, we get a continuous spectrum of KK modes with positive $m^2 > 0$. However, it is shown that only the massless chiral modes could be localized on the brane.

A. $F(\phi) = \phi$

Here, we face the difficulty that for general τ we can not obtain the function $y(z)$ in an explicit form. But we can write the potentials as a function of y:

$$\begin{aligned}
V_L(z(y)) &= \frac{1}{4}q\eta\cosh^{-2\tau}(ky)\left(-\frac{8\,e^{ky}k}{1 + e^{2ky}}\right. \\
&\quad + q\eta(\pi - 4\arctan e^{ky})^2 \\
&\quad \left. - 2k\tau(\pi - 4\arctan e^{ky})\tanh ky\right), \tag{23} \\
V_R(z(y)) &= V_L(z(y))|_{\eta\to-\eta}. \tag{24}
\end{aligned}$$

This potential for the left chiral fermions has the asymptotic behavior: $V_L(y = \pm\infty) = 0$ and $V_L(y = 0) = -kq\eta$, where $k > 0$. For $q\eta > 0$, this is in fact a volcano type potential [43, 44]. The effective potential for the left chiral fermions is shown in Fig. 1. For positive
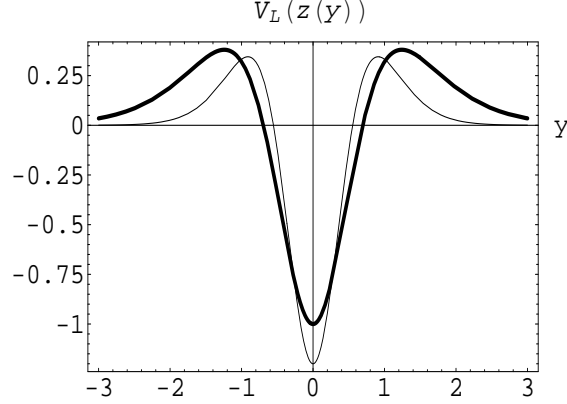
FIG. 1: The shape of the potential $V_L(z(y))$ for the case $F(\phi) = \phi$. The parameters are set to $k = 1$, $q = 1$, $\eta = 1$ and $\tau = 1$ for thick line, and $\eta = 1.2$ and $\tau = 2$ for thin line.

τ, $z(y)$ is a monotonous function, which means that the potential for arbitrary positive τ provides no mass gap to separate the zero mode from the excited KK modes.

In the following, without loss of generality, we mainly discuss the case $\tau = 1$, for which one can invert the coordinate transformation $dz = \mathrm{e}^{-A(y)}dy$, namely

$$y = \operatorname{arcsinh}(kz)/k, \tag{25}$$

and get the explicit forms of the potentials and the kink $\phi(z)$

$$
\begin{aligned}
V_L(z) &= \eta\Bigg(\frac{q^2\eta(\pi - 4\arctan \mathrm{e}^{\operatorname{arcsinh}kz})^2}{4(1 + k^2z^2)} \\
&\quad - \frac{2kq\ \mathrm{e}^{\operatorname{arcsinh}kz}}{(1 + k^2z^2)(1 + \mathrm{e}^{2\operatorname{arcsinh}kz})} \\
&\quad - \frac{k^2qz(\pi - 4\arctan \mathrm{e}^{\operatorname{arcsinh}kz})}{2(1 + k^2z^2)^{3/2}}\Bigg),
\end{aligned} \tag{26}
$$

$$V_R(z) = V_L(z)|_{\eta \to -\eta}, \tag{27}$$

$$\phi(z) = 2q\arctan \mathrm{e}^{\operatorname{arcsinh}(kz)} - \frac{\pi q}{2}. \tag{28}$$

The values of the potentials for the left chiral and right chiral fermions at $y = 0$ are given by

$$V_R(0) = -V_L(0) = kq\eta. \tag{29}$$

Both potentials have the asymptotic behavior: $V_{L,R}(z = \pm\infty) = 0$. But for a given coupling constant η, the values of the potentials at $z = 0$ are opposite. The shape of the kink $\phi(z)$

and the above two potentials are shown in Fig. 2 for given values of positive η and q. It can be seen that $V_L(z)$ is indeed a volcano type potential. Hence, the potential provides no mass gap to separate the fermion zero mode from the excited KK modes, and there exists a continuous gapless spectrum of the KK modes for both the left chiral and right chiral fermions.
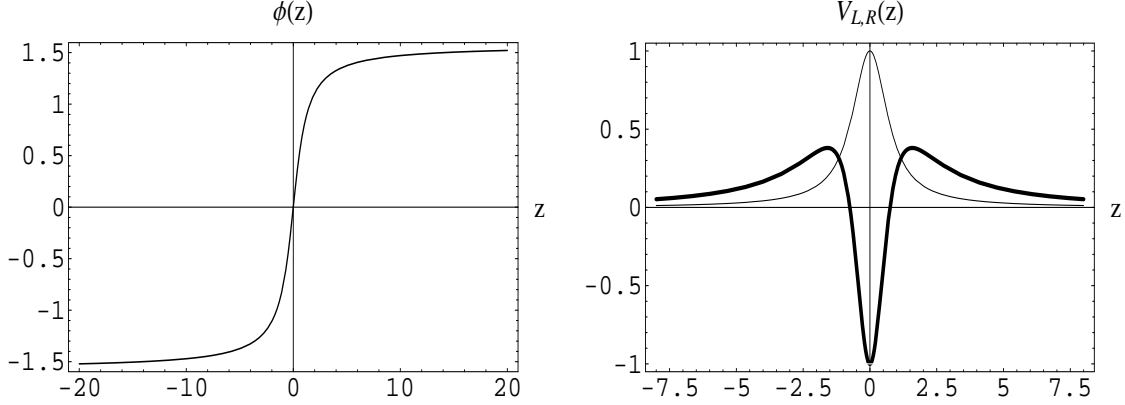

FIG. 2: The shape of the kink ($\phi(z)$ with positive k) and the potentials $V_L(z)$ (thick line) and $V_R(z)$ (thin line) for the left and right chiral fermions for the case $F(\phi) = \phi$ and $\tau = 1$ in z coordinate. The parameters are set to $k = 1, q = 1$, and $\eta = 1$.

For positive q and η, only the potential for left chiral fermions has a negative value at the location of the brane, which could trap the left chiral fermion zero mode solved from (18a) by setting $m_0 = 0$:

$$\widetilde{\alpha}_{L0}(z) = e^{2A} \alpha_{L0}(z) \propto \exp\left(-\eta \int^z dz' e^{A(z')} \phi(z')\right). \tag{30}$$

In order to check the normalization condition (19) for the zero mode (30), we need to check whether the inequality

$$\int dz \exp\left(-2\eta \int^z dz' e^{A(z')} \phi(z')\right) < \infty \tag{31}$$

is satisfied. For the integral $\int dz e^A \phi$, we only need to consider the asymptotic characteristic of the function $e^A \phi$ for $z \to \infty$. Noting that $\arctan z \to \pi/2$ when $z \to \infty$, we have

$$e^A \phi = \frac{4q \arctan e^{\operatorname{arcsinh} kz} - q\pi}{2\sqrt{1 + k^2 z^2}} \to \frac{q\pi}{2\sqrt{1 + k^2 z^2}}, \tag{32}$$

$$\int dz e^A \phi \to \frac{q\pi}{2k} \operatorname{arcsinh} kz. \tag{33}$$

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Now, the normalization condition (31) is changed to $\int dz \exp\left(-\frac{\eta q \pi}{k}\mathrm{arcsinh}kz\right) < \infty$. Hence the condition on the free parameters η and q is

$$\eta q > \frac{k}{\pi}. \tag{34}$$

In fact, we can solve the problem in y coordinate easily. In this coordinate, the condition (31) becomes

$$\int dy \exp\left(-A(y) - 2\eta \int^y dy'\phi(y')\right) < \infty. \tag{35}$$

When $y \to \infty$, we have $A(y) \to -ky$ and $\phi(y) \to q\pi/2$, and so $\left(-A(y) - 2\eta \int^y dy'\phi(y')\right) \to (k - \eta q\pi)y$. Then we can get the restriction condition (34) for localizing the zero mode of the left chiral fermions.

The zero mode (30) represents the lowest energy eigenfunction (ground state) of the Schrödinger equation (20) since it has no zeros, and it is the only one bound state. Since the ground state has the lowest mass square $m_0^2 = 0$, there is no tachyonic left chiral fermion mode. The potential (26) provides no mass gap to separate the fermion zero mode from the excited KK modes. In Fig. 3, we plot the left chiral fermion potential $V_L(z)$, the corresponding zero mode, and the massive KK modes. We see that the zero mode is bound on the brane, while the massive modes propagate along the extra dimension. Those massive modes with lower energy experience an attenuation due to the presence of the potential barriers near the location of the brane.

In the case $q\eta > 0$, the potential for the right chiral fermions is always positive, which shows that it can not trap the right chiral zero mode. But for the case of negative $q\eta$, things are opposite and only the right chiral zero mode can be trapped on the brane. For arbitrary $q\eta \neq 0$, the two potentials suggest that there is no mass gap but a continuous spectrum of KK modes with $m^2 > 0$.

B. $F(\phi) = \sin\phi$

For the case $F(\phi) = \sin\phi$, the potential as a function of y for the left chiral fermions is

$$V_L(z(y)) = -\frac{1}{2}\eta \cosh^{-1-2\tau}(ky)\Big[\eta\left(\cos(2\phi(y)) - 1\right)\cosh ky$$

$$+ 2kq\cos\phi(y) - \tau q\sin\phi(y)\sinh ky\Big]. \tag{36}$$

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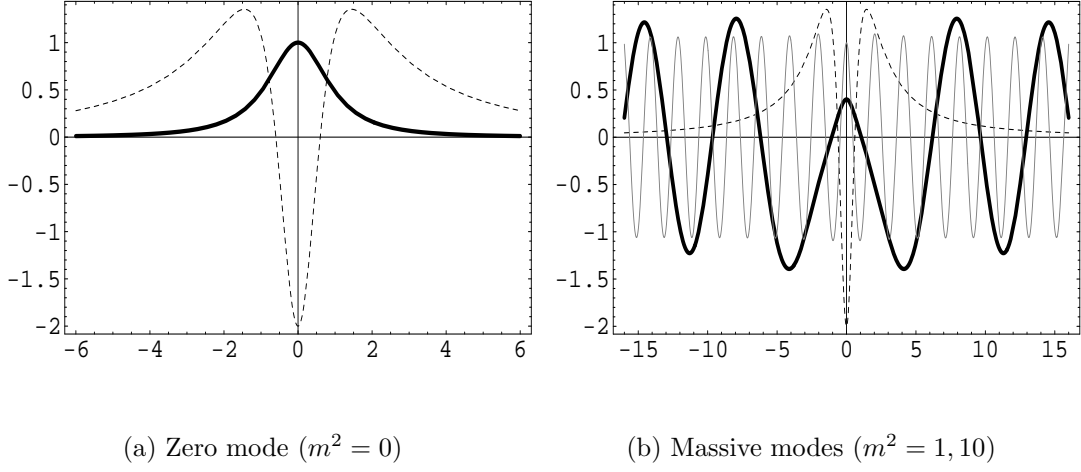(a) Zero mode ($m^2 = 0$)　　　　　　(b) Massive modes ($m^2 = 1, 10$)

FIG. 3: The shape of the potentials $V_L(z)$ (26) (dashed lines), the zero mode (30) and the massive modes for the left chiral fermions for the case $F(\phi) = \phi$. The parameters are set to $k = 1, q = 2$, and $\eta = 1$. In the right figure, we set $m^2 = 1$ and 10 for thick black line and thin gray line, respectively.

For different values of q, $F(\phi(y))$ has different behaviors, which should result in different types of the potential V_L. According to the expression of $F(\phi)$:

$$F(\phi(y)) = \sin\left(2q \arctan \mathrm{e}^{ky} - \frac{\pi q}{2}\right), \tag{37}$$

we have $F|_{y \to \infty} \to \sin \frac{q\pi}{2}$. So, when $y \to \infty$, $F(\phi(y))$ has different limits for different values of q:

$$F(y \to \infty) > 0 \quad \text{for} \quad 4n < q < 4n + 2, \tag{38}$$

$$F(y \to \infty) = 0 \quad \text{for} \quad q = 2n, \tag{39}$$

$$F(y \to \infty) < 0 \quad \text{for} \quad 4n + 2 < q < 4n + 4, \tag{40}$$

where n is an arbitrary integer. The shapes of $F(\phi(y)) = \sin \phi(y)$ for various values of q are shown in Fig. 4.

Considering that τ does not change the characteristic of the effective potentials acting on the left and right chiral fermions, we mainly focus on the case $\tau = 1$, for which one can
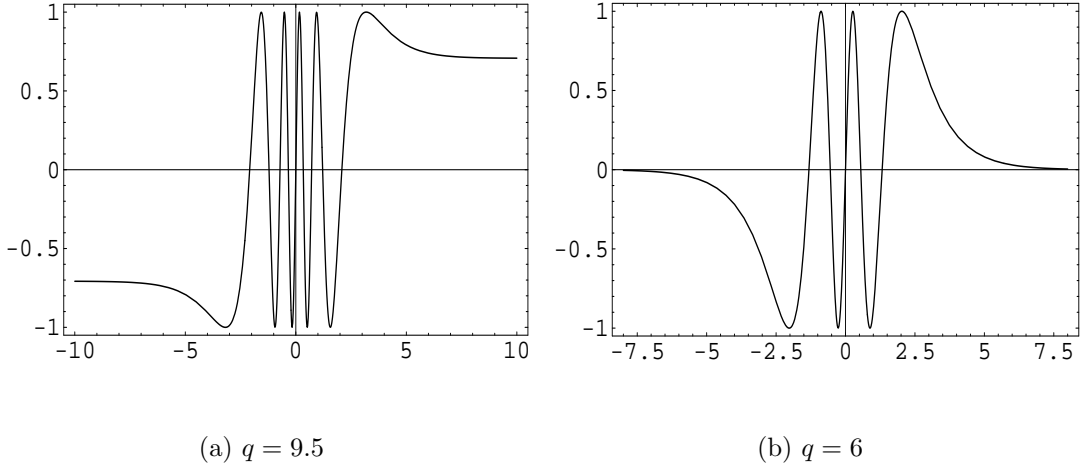
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(a) $q = 9.5$ (b) $q = 6$

FIG. 4: The shape of $F(\phi) = \sin\phi$ for various values of q in y coordinate. For $q = 9.5$, which is between $4n$ and $4n + 2$, F tends to a positive constant when $z \to \infty$ and tends to a negative constant when $z \to -\infty$. For $q = 6$, which is an even number, F tends to zero when $z \to \pm\infty$.

get the explicit forms of the potentials in z coordinate

$$
\begin{aligned}
V_L(z) &= \eta\left(\frac{k^2 z \sin\phi(z)}{(1 + k^2 z^2)^{3/2}} + \frac{\eta \sin^2\phi(z)}{1 + k^2 z^2}\right. \\
&\left. - \frac{2kq\mathrm{e}^{\mathrm{arcsinh}kz}\cos\phi(z)}{(1 + \mathrm{e}^{2\mathrm{arcsinh}kz})(1 + k^2 z^2)}\right),
\end{aligned}
\tag{41}
$$

$$
V_R(z) = V_L(z)|_{\eta \to -\eta},
\tag{42}
$$

where $\phi(z)$ is given by Eq. (28). The values at $y = 0$ are given by

$$
V_L(0) = -V_R(0) = -kq\eta.
\tag{43}
$$

Both potentials have the asymptotic characteristic: $V_{L,R}|_{z \to \pm\infty} \to 0$. But for a given coupling constant η, the values of the potentials at $z = 0$ are opposite. It can be seen from (41) and (42) that the shapes of the potentials are determined by $\sin\phi(z)$ and $\cos\phi(z)$, which depend closely on the value of q.

For general q, V_L is not any more a volcano type potential. Here we only discuss the case of positive η and q, which results in a negative potential at the location of the brane since k is positive. In order to localize fermions on the brane, we also need at least a potential barrier on each side. In fact, for $4n < q \leqslant 4n + 2$ or $q = 4n + 3$, the potential for the left chiral fermions has $n + 1$ finite positive barriers on each side, in which the last one on each side vanishes asymptotically from above. The potential is always positive at long distances,

so it can trap the zero mode. The shapes of the potential for this case are shown in Fig. 5. For $4n+2 < q \leqslant 4n+4$ but $q \neq 4n+3$, the potential for the left chiral fermions has also $n+1$ finite positive barriers on each side, but the last barrier on each side vanishes asymptotically from below. The potential is always negative at long distances, which indicates that it can not trap the zero mode. See Fig. 6 for the shapes of the potential. Hence, in order to get a potential for the left chiral fermions that can trap some fermion KK modes, we first need the following condition

$$4n < q \leqslant 4n + 2 \ (n \geq 0) \quad \text{or} \quad q = 4n + 3 \ (n \geq 0). \tag{44}$$

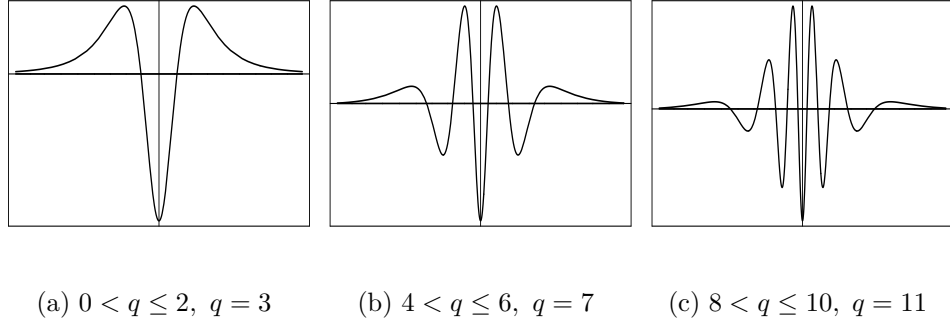(a) $0 < q \leq 2$, $q = 3$ (b) $4 < q \leq 6$, $q = 7$ (c) $8 < q \leq 10$, $q = 11$

FIG. 5: The shapes of the potential $V_L(z)$ (41) for $4n < q \leq 4n + 2$ or $q = 4n + 3$. The potential has a negative value at the location of the thick brane, and $n + 1$ finite positive barriers on each side which vanish asymptotically from above when far away from the brane.
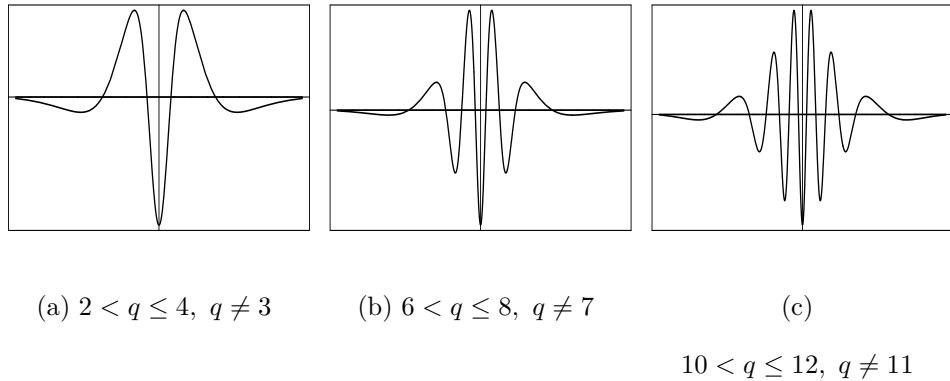

(a) $2 < q \leq 4$, $q \neq 3$ (b) $6 < q \leq 8$, $q \neq 7$ (c)

$10 < q \leq 12$, $q \neq 11$

FIG. 6: The shapes of the potential $V_L(z)$ (41) for $4n + 2 < q \leq 4n + 4$ but $q \neq 4n + 3$. The potential has a negative value at the location of the thick brane, and $n + 1$ finite positive barriers on each side which vanish asymptotically from below when far away from the brane.

Next we discuss the relation of the potential $V_R(z)$ with the parameter q. We also limit our discussion on positive η and q, which results in a positive potential at the location of the brane for $V_R(z)$. This looks like that the potential could not trap any KK modes of the right chiral fermions. But the result is opposite. For $4n < q \leqslant 4n + 2$ but $q \neq 4n + 1$, the potential for the right chiral fermions has $2n + 1$ finite barriers and $2n + 2$ finite wells, among them a positive barrier is located at the location of the brane. The potential vanishes asymptotically from below at long distances, which indicates that it can not trap the zero mode. For $4n + 2 < q \leqslant 4n + 4$, the potential has $2n + 3$ finite barriers and $2n + 2$ finite wells. For $q = 4n + 1$, the potential has $2n + 1$ finite barriers and $2n$ finite wells. For both cases, the potential vanishes asymptotically from above at long distances, which indicates that it can not trap the zero mode. Hence, in order to get a potential for the right chiral fermions that can trap some fermion KK modes, we need the following condition

$$4n + 2 < q \leqslant 4n + 4 \ (n \geq 0) \text{ or } q = 4n + 1 \ (n > 0). \tag{45}$$

This is a remarkable result which is very different from the case considered in previous subsection, where the potential for the right chiral fermions with positive η and q can not trap any KK modes because it is always positive. The shapes of the potential $V_R(z)$ for various values of q are shown in Figs. 7 and 8.
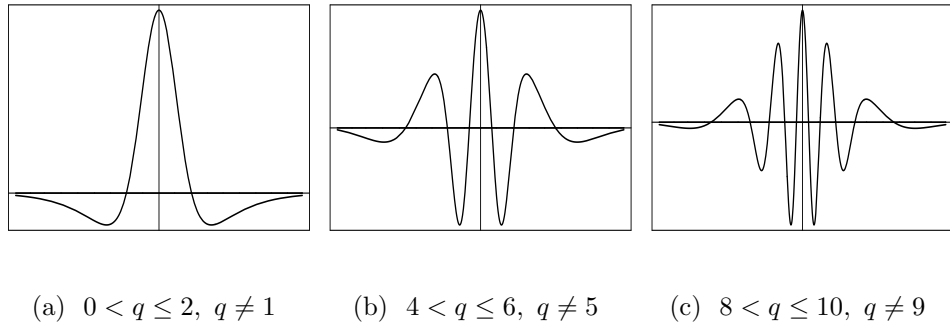

(a) $0 < q \leq 2$, $q \neq 1$ (b) $4 < q \leq 6$, $q \neq 5$ (c) $8 < q \leq 10$, $q \neq 9$

FIG. 7: The shapes of the potential $V_R(z)$ (42) for $4n < q \leq 4n + 2$ but $q \neq 4n + 1$. The potential has a positive value at the location of the thick brane, and n finite positive barriers on each side which vanish asymptotically from below when far away from the brane.

Now we examine the zero modes for the left and right chiral fermions. By setting $m_0 = 0$ and $F(\phi) = \sin \phi$, from Eq. (18) we find the left and right zero modes have the following

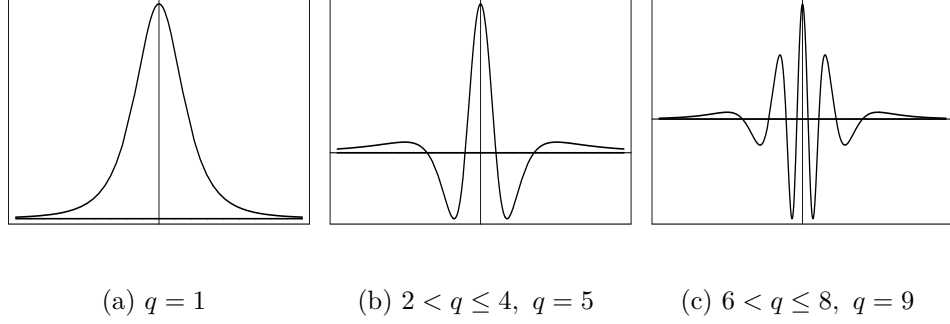(a) $q = 1$ (b) $2 < q \leq 4$, $q = 5$ (c) $6 < q \leq 8$, $q = 9$

FIG. 8: The shapes of the potential $V_R(z)$ (42) for $4n + 2 < q \leq 4n + 4$ and $q = 4n + 1$. The potential has a positive value at the location of the thick brane, as well as $n + 1$ and n finite positive barriers for $4n + 2 < q \leq 4n + 4$ and $q = 4n + 1$ respectively on each side which vanish asymptotically from above when far away from the brane.

formalized solutions:

$$\widetilde{\alpha}_{L0}(z) \propto \exp\left(-\eta \int^z dz' e^{A(z')} \sin \phi(z')\right), \tag{46}$$

$$\widetilde{\alpha}_{R0}(z) \propto \exp\left(+\eta \int^z dz' e^{A(z')} \sin \phi(z')\right). \tag{47}$$

Using the same method in previous subsection, we can obtain the restriction on the free parameters η and q from the normalization condition (19) for the zero modes (46) and (47). In y coordinate, the restriction condition for the zero modes is

$$\int dy \exp\left(-A(y) - (\pm)2\eta \int^y dy' \sin \phi(y')\right) < \infty. \tag{48}$$

When $y \to \infty$, we have $A(y) \to -ky$ and $\sin \phi(y) \to \sin \frac{q\pi}{2}$, and so $\left(-A(y) - (\pm)2\eta \int^y dy' \sin \phi(y')\right) \to (k - (\pm)2\eta \sin \frac{q\pi}{2})y$. Thus, the restriction condition reduces to

$$\pm 2\eta \sin(q\pi/2) > k \tag{49}$$

with "+" for the left fermions and "−" for the right ones. For the left chiral fermions, combining (49) with the constrain (44) coming from the effective potential V_L, the condition for localizing the zero mode is turned out to be

$$4n < q < 4n + 2 \ \ \text{or} \ \ q = 4n + 3 \ (n \geq 0),$$
$$\eta > \frac{k}{2\sin(q\pi/2)}. \tag{50}$$

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For the right chiral fermions, the localization condition of the zero mode is

$$4n + 2 < q < 4n + 4 \quad \text{or} \quad q = 4n + 1 \ (n \geq 0),$$
$$\eta > \frac{k}{-2\sin(q\pi/2)}. \tag{51}$$

Note that the values $q = 2n$ do not appear in (50) and (51). The reason is that the value of $F(\sin\phi)$ will tend to zero at long distances for $q = 2n$, which results in the non-normalizable zero modes. In Ref. [45], Melfo et al studied the localization of fermions on various scalar thick branes. They showed that only one massless chiral mode is localized in double walls and branes interpolating between different AdS_5 space-times whenever the wall thickness is keep finite, while chiral fermion modes cannot be localized in dS_4 walls embedded in a M_5 space-time. In Ref. [46], Bietenholz et al investigated fermions in a brane world of the 3-d Gross-Neveu Model and addressed in particular the question if approximate chiral symmetry can come about in a natural way under brane-type dimensional reduction. They found that a left-handed 2D fermion localized on the domain wall and a right-handed fermion localized on the anti-wall communicate with each other through the 3D bulk, and the two 2D fermions are bound together to form a Dirac fermion of mass m. This involves a hierarchy problem with respect to the fermion mass.

For arbitrary $q\eta > 0$, the two potentials suggest that there is no mass gap but a continuous spectrum of KK modes. In Fig. 9, we plot the massless and massive KK modes for the left chiral fermions. It can be seen that the zero mode is bound on the brane if the condition (50) is satisfied. The massive modes with lower energy (especially the zero mode) experience an attenuation due to the series of potential barriers near the location of the brane.

To close this section, we make some comments on the issue of the localization of fermions. Localizing the fermions on branes or defects requires us to introduce other interactions besides gravity. More recently, Volkas et al had extensively analyzed localization mechanisms on a domain wall. In particular, in Ref. [47], they proposed a well-defined model for localizing the SM, or something close to it, on a domain wall brane. There are some other backgrounds, for example, gauge field [48], supergravity [49, 50] and vortex background [51, 52, 53, 54], could be considered. The topological vortex coupled to fermions may result in chiral fermion zero modes [55].
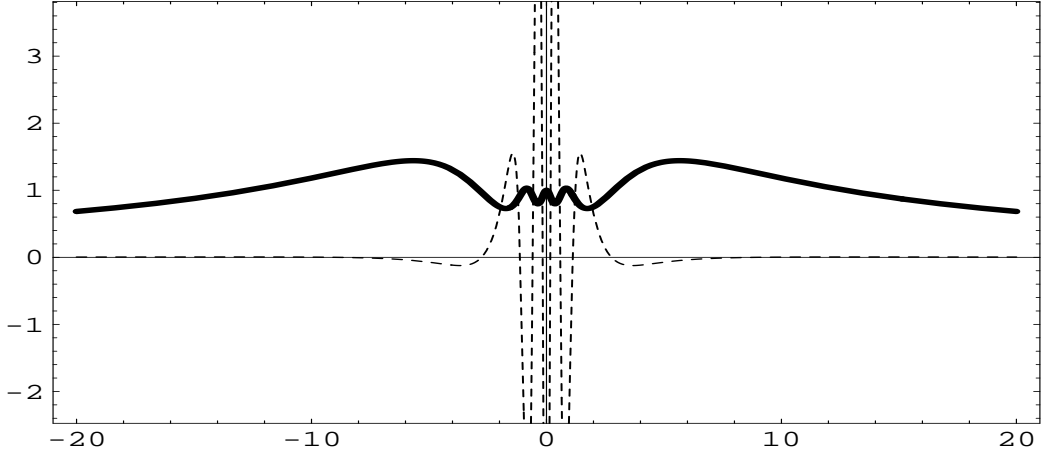
(a) Zero mode ($m^2 = 0$, $q = 9$)
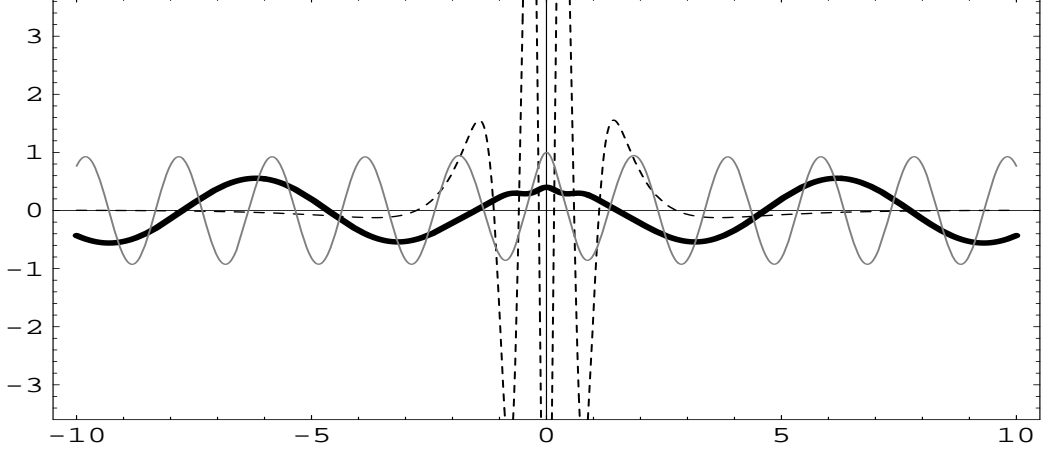
(b) Massive modes ($m^2 = 1$ for thick black line, $m^2 = 10$ for thin gray line, $q = 9$)

FIG. 9: The shape of the potentials $V_L(z)$ (41) (dashed lines), the zero mode (46) and the massive modes for the left chiral fermions for the case $F(\phi) = \sin\phi$. The parameters are set to $k = 1, q = 9$, and $\eta = 1$.

IV. DISCUSSIONS

In this paper, by presenting the shapes of the mass-independent potentials in the corresponding Schrödinger equations, we have reinvestigated the possibility of localizing spin half fermions on a thick brane for two kinds of kink-fermion couplings. It is shown that, without scalar-fermion coupling, there is no bound state for both the left and right chiral

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fermions. Hence, in order to localize the massless and massive left or right chiral fermions on the brane, some kind of Yukawa coupling should be introduced.

For the Yukawa coupling $\eta\bar{\Psi}\phi\Psi$, only one of the potentials for the left and right chiral fermions has a finite well at the location of the brane and a finite barrier on each side which vanishes asymptotically. It is shown that there is only one single bound state (zero mode) which is just the lowest energy eigenfunction of the Schrödinger equation for the corresponding chiral fermions. When the condition $\eta q\pi > k$ is satisfied, the zero mode is normalizable.

For the scalar-fermion coupling $\eta\bar{\Psi}\sin\phi\Psi$ with $q > 0$ and $\eta > 0$, the potential for the left chiral fermions has a finite well at the location of the brane as well as a series of finite positive barriers on each side, and vanishes asymptotically from above or below when far away from the brane. Under the condition (50), there exists a bound and normalizable left chiral fermion zero mode.

It is worth to point out that, under the condition $q > 0$ and $\eta > 0$, for the usual coupling $\eta\bar{\Psi}\phi\Psi$, the potentials for the left and right chiral fermions have very different shapes and only the left fermion zero mode could be localized. However, for the coupling $\eta\bar{\Psi}\sin\phi\Psi$, the potentials for the left and right chiral fermions have similar shapes and the right fermion zero mode also could be localized on the brane under the condition (51). The reason is that, although the potential for the right chiral fermions has a positive value at the location of the brane, it has some wells and a series of positive barriers near the brane, which ensures that it can trap the right chiral fermion zero mode on the brane.

Since the potentials for both scalar-fermion couplings vanish asymptotically when far away from the brane, all values of $m^2 > 0$ are allowed, and there exists no mass gap but a continuous gapless spectrum of KK states with $m^2 > 0$. The massive KK modes asymptotically turn into continuous plane waves when far away from the brane [2, 21], which represent delocalized massive KK fermions.

Acknowledgement

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